HELSINKI SCHOOL OF ECONOMICS (HSE)
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GARCH MODELLING OF INTEREST RATE AND EXCHANGE RATE VOLATILITY

Analysis of US T-Bill, Euribor and US dollar-Euro exchange rate

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HELSINKI SCHOOL OF ECONOMICS | ABSTRACT
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GARCH MODELLING OF INTEREST RATES AND EXCHANGE RATES

PURPOSE OF THE STUDY

The objective of this Thesis is to model the volatility of US interest rate and Euribor interest rate time series as well as US dollar – Euro exchange rate time series by using GARCH models. We employ so called error correction model where we test the stationarity of time series and then estimate the level model by using ARMA modelling. Then the residuals of the level models are examined for heteroskedasticity and the most suitable GARCH models are being applied. I use the GARCH models for forecasting the volatility. Literature review covers the previous research done in this field.

DATA

Data consist of daily observations of 3 month US Treasury Bill (T-Bill) interest rate, 3 month Euribor interest rate US dollar – Euro exchange rate. The starting date of the data is 31.12.1998 and the end date is 31.12.2007.

RESULTS

The findings show that all the time series examined are non-stationary, meaning we have to differentiate the time series, i.e. investigate the returns. We found that the most suitable level model for Euribor is ARMA(2,2). The residual terms prove not to feature heteroskedasticity, so GARCH model is not applied. The most suitable ARMA model for US T-Bill is ARMA(2,2) and the corresponding GARCH model is GARCH(2,3). However, the GARCH model did not prove to be stable. Furthermore, we found that the most suitable level model for USD-Euro exchange rate is ARMA(2,3) and the GARCH model GARCH(3,2).

KEYWORDS

Volatility modelling, GARCH, ARMA, heteroskedasticity, time series analysis

HELSINGIN KAUPPAKORKEAKOULU

Rahoituksen Pro Gradu-tutkielma

Tuomas Gynther

TIIVISTELMÄ

30.6.2008

KORKOJEN JA VALUUTTAKURSSIN VOLATILITEETIN GARCH-MALLINNUS

TUTKIMUKSEN TARKOITUS

Tämän tutkimuksen tarkoituksena on mallintaa Yhdysvaltain kolmen kuukauden koron, kolmen kuukauden Euriborin ja Yhdysvaltain dollarin ja Euron välisen valuuttakurssin volatiliteettia käyttäen GARCH-mallia. Mallinnuksessa hyödynnämme virheenkorjausmallia, jossa ensin tarkastelemme aikasarjojen stationaarisuutta. Tämän jälkeen estimoimme aikasarjoille tasomallit. Tasomallien tuloksena saaduille jäännöstermeille estimoimme GARCH-mallin, mikäli jäännöstermeissä on havaittavissa heteroskedastisuutta. Löydettyä GARCH-mallia käytämme volatiliteetin ennustamiseen. Kirjallisuuskatsaus käy läpi alan viimeaikaista kehitystä ja tutkimustuloksia.

AINEISTO

Aineisto koostuu Yhdysvaltain kolmen kuukauden T-Billin aikasarjasta, kolmen kuukauden Euriborin aikasarjasta sekä Yhdysvaltain dollarin ja Euron välisen valuuttakurssin aikasarjasta. Käytämme päivähavaintoja 31.12.1998 – 31.12.2007 väliseltä ajalta.

TULOKSET

Tutkimushavaintojemme mukaan kaikki aikasarjat ovat ei-stationaarisia. Tämä tarkoittaa, että meidän täytyy differoida aikasarjat eli tutkia niiden tuottoja. Mallinnuksen mukaan sopivin tasomalli Euriborille on ARMA(2,2). Sitä vastoin testit eivät osoittaneet merkkejä jäännöstermien heteroskedastisuudesta eli GARCH-mallia ei sovelleta. Sopivin tasomalli T-Billille on ARMA(2,2) ja GARCH-malli GARCH(2,3). Tämä GARCH-malli ei kuitenkaan osoittautunut stabiiliksi. Valuuttakurssille sopivin tasomalli on ARMA(2,3) ja GARCH-malli GARCH(3,2).

AVAINSANAT

Volatiliteetin mallinnus, GARCH, ARMA, heteroskedastisuus, aikasarja-analyysi

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1 INTRODUCTION

Exchange rates and interest rates play a crucial role in the international economy. Currency markets are the biggest markets in the world of finance by volume. The exchange rates are largely driven by interest rate differentials between the corresponding countries. Furthermore, not only the levels of exchange rates are important but the volatility is as well for both academics and practitioners. Exchange rate volatility is important in the pricing of foreign exchange (FX) options. Volatility modelling and especially the GARCH model have become important tools in economics and finance. It is widely recognized that volatility is time-varying and periods of high volatility tend to cluster. This can be captured by using autoregressive conditional heteroskedasticity (ARCH) models, first introduced by Engle (1982). Later these models were extended to generalized ARCH (GARCH) by Bollerslev (1986).

The Euro-zone financial markets have gained importance in the global economy since the introduction of Euro currency. They have also challenged the US financial markets' position as the most important market in the world. Hence, it is worth to examine the relationships between those two markets. We do it in this study by taking a closer look at the interest rate and exchange rate volatilities. We use time series analysis in order to model and find out relations between and to determine the long-run behaviour of three time series, namely two interest time series, i.e. 3-month euribor, 3-month US T-bill, and Euro-US dollar exchange rate and to predict their future volatilities. The literature part covers the most relevant aspects related to time series analysis in overall and illustrates the possible problems.

2 LITERATURE REVIEW

In this section we cover the most relevant aspects and concepts in time series analysis and provide the latest findings in the field of GARCH modelling and research.

2.1 About time series analysis models

Granger and Poon (2001) state that the financial market volatility is an important input for investment, option pricing and financial market regulation. They compare different volatility forecasting findings published in the last decade. The authors start with describing time series (or historical price) techniques for volatility forecasting. They state that the simplest historical price model is the Random Walk (RW) model. They argue that if change in volatility is i.i.d., then volatility forecast can be based on any period in the past although one often uses t-1 value to predict time t volatility. Extensions from this idea include Historical Average (HA) method, Moving Average (MA) method, the Exponential Smoothing (ES) method and the Exponentially Weighted Moving Average (EWMA) method.

The ARCH family models are more sophisticated group of time series models. The ARCH(q) model relates time t volatility to q past squared returns, with no predetermined relationships between the q dependencies. All ARCH class models assume variance stationarity, and thus perform poorly when the series is nonstationary. GARCH(p,q) permits additional dependencies on p lags of past estimated volatility. Empirical findings suggest that GARCH is a more parsimonious model than ARCH. EGARCH models volatility in logarithmic form, so there is no need to impose estimation constraint in order to avoid negative variance because it is the logarithm of variance at t that is formulated. If the parameters are appropriately conditioned, this specification captures the stylised fact that negative shocks lead to higher volatility in the subsequent period than that triggered by a positive shock.

Granger and Poon state that the simpler methods, including EWMA method, are not adaptive, meaning that their volatility structure does not respond readily to returns shocks. GARCH is more adaptive: Sources of volatility are separated into volatility that is due to past shocks and volatility carried forward due to persistence. In overall, simpler methods tend to provide larger volatility forecasts than the more sophisticated models.

Granger and Poon point out that there are at least three facts in the volatility modelling literature that are not captured by the ARCH/GARCH models. Firstly, the standardised residuals from ARCH/GARCH model still display large kurtosis, meaning that conditional heteroskedasticity alone could not account for all the tail thickness. Secondly, the null hypothesis of a unit root in variance is not rejected by several authors using different sets of stock market data. Thirdly, GARCH effect disappears once large shocks are controlled for. Forecasting performance of GARCH model can be substantially improved by removing the additive outliers. This raises the question whether all the extremes or outliers should be removed, because the level of volatility associated with a stock market crash in not likely to be repeated every day, but the effect of the increase in volatility will be felt over a considerable period of time. There is one possible solution for tackling changing volatility level and persistence. That is to use regime switching (RS) model, where volatility persistence can take different values depending on whether it is in high or low volatility regimes.

2.2 Cointegration effect

Granger and Yoon (2002) identified possibly hitherto unnoticed cointegrating relationships among integrated components of interest rate series for which no standard cointegration is found. They argue that if the components are cointegrated the data are said to have hidden cointegration (i.e. the variables respond together only to positive or negative shocks but not for both). According to Granger and Yoon, standard cointegration is a special case of hidden cointegration. Furthermore, hidden cointegration is a simple example of nonlinear cointegration. Although the data series are not cointegrated in the conventional sense, it is still possible for them to have hidden cointegration, which would help better understand their dynamic relationships and produce improved forecasts

They state that it is customary to investigate the existence of cointegrating relationships among integrated (often called nonstationary) economic variables before estimating parameters or testing hypothesis. If the data are cointegrated, error-correction models are estimated, otherwise VAR models are estimated in first differences. To test hidden cointegration, the authors define so called crouching error correction models of cointegrated components.

DeGennaro, Kunkel and Lee (1994) investigated the relationship among interest rates on the long term government bonds of five industrialized countries (Canada, Germany, Japan, the United

Kingdom, the United States). They applied unit root and cointegration (Johansen-Juselius) tests which confirmed the presence of exactly one unit root. These tests are important for both modelling and forecasting purposes.

DeGennaro & al. find that if each variable in a system of interest rates is stationary, then the system is best modelled using some set of relevant variables in conjunction with time series specification. On the other hand, if each series contains an independent unit root, one may consider a vector-autoregression (VAR) in differences. But, if a nonstationary system is cointegrated, a VAR is misspecified and the error correction model of Engle and Granger provides better forecasts.

They found only very little evidence of cointegration among the five long-term interest rate series. That means that when modelling or forecasting this set of international interest rates, researchers may need to assume separate sets of fundamentals because there appears to be no common long-run stochastic trend binding these series together. Also, the data may have to be differentiated in order to achieve stationarity. Thus, an error correction model may not be appropriate because these models utilize the information in a long-run equilibrium relationship. These findings were contrary to previous studies.

Georgoutsos and Kouretas (2001) developed a testing methodology of two interest rate determination theories. The empirical research on the determination of the interest rates appears to have been conducted on two lines of research. First, the interest rate parity doctrine with market efficiency hypothesis which implies that the yields of domestic and foreign assets can differ only by the expected change in the price of foreign exchange, and second, the expectations theory of the term structure.

Their analysis is conducted within the context of cointegration and therefore they examine the existence of a long-run relationship among ten interest rates from eurodollar and euromark markets with maturity ranging from seven days to one year. They provide a new analysis for the determination of the order of integration of the variables. In order to avoid misspecified model they employ the Full Information Maximum Likelihood multivariate cointegration methodology (Johansen-Juselius). Secondly, they identify whether there are several cointegrating vectors (since in a multivariate framework). Thirdly, given that they are able to find at least one statistically significant cointegrating vector, they examine the stability of the long-run relationships through time (Rank test).

Georgoutsos and Kouretas found nine statistically significant cointegrating vectors among the system of ten interest rates. Also, given that more than one long-run relationships were found they imposed independent linear and homogeneous restrictions on the system and the joint structure of the expectations theory and the interest rate parity could not be rejected which implies that their framework is valid to study the interdependence of monetary policy in an integrated scheme. Thirdly, the parameter stability tests show that the cointegration results are sample independent.

Christoffersen and Diebold (1997) write that cointegration implies restrictions in the low-frequency dynamic behaviour of multivariate time series. They argue that imposition of cointegrating restrictions has immediate implications for the behaviour of long-horizon forecasts and, if fact, will produce superior long-horizon forecasts. If the variables are cointegrated their values are linked over the long run and when this information is imposed it will produce substantial improvement in forecast over long horizons.

Christoffersen and Diebold state that the forecast enhancement from exploiting cointegration stems from using information in the current deviations from the cointegrating relationships. They add, however, that although the current value of the error correction term provides information about the likely near-horizon evolution of the system, it seems unlikely that it provides information about the long-horizon evolution of the system because the long-horizon forecast of the error correction term is always zero (covariance stationary with zero mean). From this perspective, it is unlikely that cointegration could be exploited to improve long-horizon forecasts.

Christoffersen and Diebold provide a precise characterization of the implications of cointegration for long-horizon forecasting. They show that imposing cointegration does not improve long-horizon forecast accuracy when forecasts of cointegrated variables are evaluated using the standard trace mean squared error (MSE) ratio but it helps in short-horizon forecasting.

Bevilacqua and Daraio (2001) studied the effects of replacing the consumer price index (CPI) with the wholesale price index (WPI) in the cointegrating international parity relationships found by Juselius and MacDonald. The Purchasing Power Parity (PPP) suggests that the exchange rate is determined by the level of prices. On the other hand, the Uncovered Interest Parity (UIP) claims that the exchange rate is determined by the spread between the interest rates in two countries.

The economic theory assumes that both PPP and UIP are stationary relations, which makes it very problematic to solve the issue. Namely, empirical evidence shows that both are nonstationary in the short and medium-long run. Bevilacqua and Daraio investigated the PPP and UIP relations by modelling them jointly. They argue that because PPP and UIP behave in nonstationary way, it is possible to investigate the cointegration relations between the two parities, i.e. the stationary long run relations between pseudo random walks that share common trends.

The joint modelling of international parity conditions produces stationary relations showing an important interaction between the goods (via the PPP) and the capital markets (via the UIP).

2.3 Stationarity

Bidarkota (1996) studied whether the use of information on the past history of the nominal interest rate and inflation entail improvements in forecasts of the *ex ante* real interest rate over its forecasts obtained from using just the past history of the realized real interest rates. He set up a univariate unobserved components model for the realized real interest rates and a bivariate model for the nominal rate and inflation which imposes cointegration restrictions between them and made comparative analysis between those two models.

Bidarkota found that the error-correction model provides more accurate one-period ahead forecasts of the real rate within the estimation sample whereas the unobserved components model yields forecasts with smaller forecast variances. Also, in the post-sample period the forecasts from the bivariate model are not only more accurate but also have tighter confidence bounds than the forecasts from the unobserved components model.

According to Bidarkota, empirical evidence fails to reject a unit root in the nominal interest rate but there seems to be some controversy regarding stochastic non-stationarity of the inflation rate. Economic theory would tell that it might be reasonable to think of the real interest rate as being stationary in any real world economy. This would suggest a cointegrating relationship between the nominal interest rate and the inflation rate, with the real rate being the cointegrating combination.

Valle (1998) studied the interdependence of Latin American and U.S. stock markets using cointegration analysis. He states that in an integrated world equity market, individual stock prices are expected to have long-run relationships. Furthermore, multivariate long-run relationships between the individual stock prices indicate the presence of common stochastic trends among the indices.

Valle tested the existence of common stochastic trends by using cointegration test developed by Johansen (Johansen's maximum likelihood method, which uses a vector error correction model). Cointegration implies that non-stationary time series such as stock prices move stochastically together towards some long-run stable relationship. Valle focuses on first order nonstationary integrated processes, i.e. I(1) processes, which require first differences to become weakly stationary. Valle tests the presence of a unit root by using the Augmented Dickey-Fuller-test (ADF). If one suspects that a structural change has occurred (leading to DF being biased towards the nonrejection of a unit root), one should split the sample into two parts and use DF tests on each part. This procedure reduces the degrees of freedom for each regression.

Valle concludes that the cointegration tests indicate a stationary long-run relationship between the stock indices. He argues that market's degree of development and co-operation among themselves play an important role.

Lanne (2002) states that according to empirical studies, US inflation and nominal interest rates, as well as the real interest rate, can be described as unit root processes. The results imply that nominal interest rates and expected inflation do not move one-for-one in the long run, which is not consistent with the theoretical models. Previous research has shown that the three-month T-Bill rate and inflation share a common nonlinear component that explains a large part of their persistence. On the other hand, the real interest rate is devoid of this component, indicating one-for-one movement of the nominal interest rate and inflation in the long run and thus stationarity of the real interest rate.

Lanne introduces a nonlinear bivariate mixture autoregressive model for the nominal interest rate and inflation which seems to fit quarterly U.S. data reasonably well. If this kind of nonlinearity lies behind the observed persistence of these time series, there may exist a linear combination of them devoid of the nonlinearities. Furthermore, if the real rate is such a linear combination, then this can be interpreted in favour of one-for-one movement of the nominal interest rate and expected inflation in the long run.

Lanne's model were able to capture the nonlinearities in the system and the model's good fit to the data suggests that the time series are stationary and the apparent nonstationarity implied by unit root tests is brought only by shifts in the level and conditional variance. Also, the model were not able to reject that the two variables share a common nonlinear component such that the real rate is devoid of nonlinearities in the level, which suggests stationarity of the real rate contrary to previous studies but it is in accordance with economic theory. Lanne concludes that the finding of a common nonlinear component in the nominal interest rate and inflation is a long-run phenomenon, i.e. the variables move hand-in-hand only in the long run.

2.4 ARCH and GARCH modelling

In their seminal paper (2005), Andersen, Bollerslev, Christoffersen and Diebold (ABCD hereafter) provided a selective survey of the most important theoretical developments and empirical insights from the field of volatility forecasting. ABCD start by arguing that the basic GARCH model assumes that positive and negative shocks of the same absolute magnitude will have the same identical influence on the future conditional variances. ABCD state that, in contrast, the volatility of aggregate equity index return has been shown to respond asymmetrically to past negative and positive return shocks, with negative returns resulting in larger future volatilities. This asymmetry is also known as a "leverage" effect. On the other hand, it is widely agreed that financial leverage alone can not explain the magnitude of the effect.

ABCD write that the asymmetry has also been attributed to a "volatility feedback" effect, which means that heightened volatility requires an increase in the future expected returns to compensate for the increased risk, i.e. necessitating a drop in the current price to go along with the initial increase in the volatility. There are at least three GARCH formulations for describing this type of asymmetry, namely the Threshold GARCH (TGARCH) models, the Asymmetric GARCH (AGARCH) models and the Exponential GARCH (EGARCH).

According to ABCD, all the GARCH models mentioned above imply that the shocks to the volatility decay at an exponential rate. This kind of exponential decay typically works well when forecasting over short horizons. However, there have been studies, which show that the autocorrelations of squared and absolute returns decay at a much slower hyperbolic rate over longer lags. This means that better long term forecasts may be obtained by taking this feature into account.

In the literature, several fractionally integrated GARCH (FIGARCH) models have been introduced to achieve this goal.

Ferreira (1999a) estimated stochastic volatility models of the short-term interest rate for Germany and France between 1981 and 1997. His focus is on the parameterisation of the interest rate volatility and the starting point is the level effect but considers a class of models, which allows volatility to depend on both interest rate level and innovations.

Ferreira also applies time varying volatility models (GARCH) in which volatility is assumed to follow a separate stochastic process (volatility is a function of the information shocks). He argues that the main problem with the GARCH specification is that interest rate levels do not have direct impact on the volatility. Furthermore, GARCH models allow negative interest rates, which is not possible for nominal rates.

Ferreira concludes that the GARCH-Level model is the best specification for the German short-term interest rate allowing for both level and conditionally heteroskedasticity effects. On the other hand, there is some evidence that models with mixed level and news effects do not outperform models with only news effect in terms of out-of-sample forecasting performance.

In his other study, Ferreira (1999b) compared the in-sample and out-of-sample forecasting performance of the spot interest rate volatility using French and German short-term interest rates between 1981 and 1997. He states that for a one-week horizon the model with the best fit does not have the biggest forecasting power.

Andrews and Kim (2003) addressed the problem of cointegration breakdown over a short period of time. The breakdown period may occur at the end of sample, the beginning of the sample or somewhere in between. Such a breakdown in a cointegrated relationship may be due to some recent event, for example, a currency crisis, productivity slowdown, policy regime shift or war. They argue that the breakdown may be due to a shift in the cointegration vector or due to a shift in the errors from being I(0) to being I(1). Andrews and Kim performed end-of-sample cointegration breakdown tests. In the model, they use regressors that are taken to be arbitrary linear combinatios of deterministic, stationary and integrated random variables.

Lanne and Saikkonen (2005) studied new nonlinear GARCH models mainly designed for time series with highly persistent volatility. They state that for such series conventional GARCH models have often proved unsatisfactory because they tend to exaggerate volatility persistence and exhibit poor forecasting ability. In their empirical applications they use two exchange rate returns series, namely German mark and Japanese yen against the U.S. dollar, and they are able to show that the new models can be superior to conventional GARCH models especially in longer term forecasting.

Lanne and Saikkonen write that there are several alternative GARCH type models that attempt to take volatility persistence into account. Hamilton and Susmel (1994) and Cai (1994) presented the regime-switching ARCH models and the fractionally integrated GARCH (FIGARCH) model was presented by Baillie, Bollerslev and Mikkelsen (1996). In their own study they employ a nonlinear alternative of the conventional GARCH model which belongs to the family of smooth transition GARCH (STGARCH). The model states that the volatility persistence can depend on the level of conditional volatility in the previous period as well as on the size of the shock.

Bauwens, Laurent and Jeroen (2004) surveyed the most important developments in multivariate ARCH-type modelling. They state that it is widely accepted that financial volatilities move together over time across assets and markets. Recognizing this feature through a multivariate modelling leads to a more relevant empirical models than working with separate univariate models. That may open the door to better decision tools in various areas, for example asset pricing, portfolio selection, option pricing and risk management.

Bauwens et al. write that the most obvious application of multivariate GARCH (MGARCH) models is the study of the relations between the volatilities and co-volatilities of several markets. This way one can study whether the volatility of a market is leading the volatility of other markets.

Klaassen (1998) studied the ways to improve GARCH volatility forecasts. In their paper they show that GARCH forecasts are too variable. Klaassen argue that it is well-known that GARCH models often imply a high volatility persistence of individual shocks. For example, Hamilton and Susmel (1994) came to the conclusion that a shock this week will have a nonnegligible effect on the variance a year later. This means that GARCH models are unable to capture the fact that shocks sometimes have the effect of "relieving pressure" on the system. In other words, a shock is followed by a period of low instead of high volatility.

Klaassen tries to improve the forecasting power of GARCH model by extending the model such that this weakness disappears. Their model contains an extra source of volatility persistence by distinguishing two regimes with different volatility levels. This source of volatility persistence added to standard GARCH models means that individual shocks can but need not to be persistent. The resulting regime-switching GARCH model yields better volatility forecasts both in-sample as well as out-of-sample. According to Klaassen, structural changes in the variance process can originate from changes in economic policy, such as the change in U.S. monetary policy in 1979 when the Federal Reserve reduced the money growth rate to bring down inflation and to stop the dollar's fall. Also, sudden shifts may result from more exogenous changes in the economic environment such as the OPEC oil crisis.

Klaassen points out a potential drawback of regime-switching ARCH models. That is that only ARCH models are allowed within a regime, not GARCH models. This is not only a theoretical disadvantage but it is also important from the practical point of view, too. This is because some series are characterized by long persistence of shocks within a regime. Klaassen writes that one GARCH term can capture such long persistence much more parsimoniously than a large number of ARCH terms. If the long persistence is neglected it may result in even worse volatility forecasts than those generated by single-regime GARCH model.

Dacorogna et al. (1997) studied how to improve the short term forecasting performance of the ARCH models. Firstly, they proposed a new formulation of the HARCH (heterogeneous ARCH) process for high frequency data. They found that volatility memory seems quite short-lived when measured with high-frequency data while it seems long-lived when measured with daily or lower frequency data.

Dacorogna et al. state that, in general, ARCH-type models are able to significantly predict the realized hourly short-term volatility out-of-sample with a limited optimization effort. In addition, models including volatility measured at temporal resolutions as in HARCH outperforms those that do not consider this effect. According to Dacorogna et al. this is an evidence of the market heterogeneity and it also emphasizes the need of high frequency data to properly analyze the financial markets.

2.5 Results of previous volatility modelling studies

Radha and Thenmozhi write that forecasting interest rates is of a great concern for financial researchers, economists and practitioners in the fixed income markets. They argue that movements in interest rates have important implications for the economy's business cycle and are crucial to understanding financial developments and changes in the economic policy. Furthermore, timely forecasts of interest rates can provide valuable information to financial market participants and policymakers. In addition, forecasts of interest rates can also help to reduce interest rate risk face by individuals and firms. And for central banks, forecasting interest rates is very useful in assessing the overall impact of its policy changes and taking appropriate corrective action.

Radha and Thenmozhi state that exchange rates were found to follow a long-term trend with short-term fluctuations. To capture this trend many authors had used ARIMA (autoregressive integrated moving average) models have been used for forecasting the exchange rate.

Radha and Thenmozhi write about a study by Yu (2002) where he evaluated the performance of nine alternative models like Random Walk, Moving average, ARCH, GARCH and stochastic volatility models for forecasting New Zealand's stock market volatility. He found that the stochastic model provided the best performance among models. Also, ARCH-type models seemed to perform well or badly depending on the form chosen. Furthermore, GARCH(3,2) was found to be the best within the ARCH family but on the other hand it was sensitive to the choice of assessment measure.

The purpose of their study was to develop an appropriate model for forecasting the short term interest rates, i.e. commercial paper rate, implicit yield on 91 day Treasury bill, overnight MIBOR rate and call money rate. They used univariate models, namely Random Walk, ARIMA, ARMA-GARCH and ARMA-EGARCH, for the forecasting purposes. Their data covered a six-year period starting from 1999.

Radha and Thenmozhi studied the log returns, i.e. $\ln(y_t) - \ln(y_{t-1})$ for forecasting the short term interest rates. They performed Augmented Dickey-Fuller test for stationarity of time series. To model the level model, they determined the AR(p) part by using partial autocorrelation function (PACF) and the MA(q) part was determined by using autocorrelation function (ACF), which indicates the significant lags for the AR and MA terms. They identified the number of lagged terms to be included in the model by the minimum value of AIC (Akaike Information Criterion) and SBC criteria.

The authors found that all the time series studied were stationary. They also found that the commercial paper rate depended only on its past three values (AR part) and implicit yield on 91 day Treasury bill depended on the previous two weeks returns. The corresponding MA term lags were zero for commercial paper and one for Treasury bill.

Radha and Thenmozhi tested the residuals of the ARMA models for ARCH effects and found that residuals of all time series suffer from ARCH effect. This means that the ARMA model is a good fit for the time series, also, but one has to use a GARCH model to better capture the heteroskedasticity.

Radha and Thenmozhi found that interest rate time series have a volatility clustering effect and hence GARCH models are more appropriate to forecast than the other models. ARIMA-EGARCH model was found to be the most appropriate for commercial paper rate, while for implicit yield 91 day Treasury bill, overnight MIBOR rate and call money rate ARIMA-GARCH model is the most appropriate for forecasting.

Hansen and Lunde (2004) compared the forecasting performance of 330 ARCH-type models in their ability to describe the one-day-ahead conditional variance. Their data set consists of daily Deutch mark – US dollar exchange rate observations as well as daily IBM return observations. The models were evaluated out-of-sample using six different loss functions where the realized variance is substituted for the latent conditional variance.

Hansen and Lunde argue that, initially, it was common to substitute the squared return for the unobserved conditional variance in out-of-sample evaluations of volatility models. They state that this has resulted in a poor performance, which instigated a discussion of the practical relevance of volatility models. Since then it's been showed by Andersen and Bollerslev (1998) that the poor performance could be explained by the fact that the squared return is a noisy proxy for the conditional variance. By substituting the realized variance instead of the squared return the volatility models perform quite well.

Hansen and Lunde's key finding was that none of the more sophisticated models outperformed GARCH(1,1) in the analysis of exchange rates, whereas the GARCH(1,1) is clearly inferior to models that can accommodate a leverage effect in the analysis of IBM returns.

Hongyu and Zhichao (2006) examined the financial volatility with the data from Chinese stock market. Their paper explores number of linear and GARCH-type models for forecasting the daily volatility of two equity indices. They started their analysis by testing the hypothesis of a unit root in the price process. They found the price process to be non-stationary, but when they subjected the logarithmic returns data to the Augmented Dickey-Fuller test they could reject the unit root hypothesis with a confidence level of more than 99%, i.e. the returns were stationary.

The authors conclude that the relative accuracies of various models are sensitive to the measure used to evaluate them. However, the Random Walk model was found to be the worst performing method for forecasting the one-day-ahead volatility.

Chortareas et al. (2007) forecasted volatility of high frequency Euro exchange rates, Euro-US dollar exchange rate among other rates. They tested the forecast performance of six models including both traditional time series volatility models and the realized volatility model.

Chortareas et el. Found that the FIGARCH model surpassed all the other models. The authors state that this finding was surprising because FIGARCH has seldom been regarded as a competitive model in high frequency application. They argue that the success may be due to its ability to capture both the long memory and volatility clustering properties, which are the two most important characteristics of volatility.

The authors also write that the intraday GARCH model performed fairly well in forecasting. This finding was contrary to the usual prejudice that the traditional daily volatility model can not fit the high frequency data framework.

3 CONCEPTS

In this section we present the most common and relevant concepts in time series analysis. These are from Helsinki School of Economics publications by Kahra and Kanto (1999).

Difference equation is an expression to relate a variable y_t to its previous values.

First-order difference equation mean that only the first lag of the variable appears in the equation, second-order difference equation, on the other hand, has two lagged values etc.

3.1 Concept of stationarity, stationarity tests and hypothesis

Shocks of stationary series are always temporary: with time the effect of a shock disappears and the series revert to the mean level. Nonstationary series, instead, have permanent components. The mean and variance of nonstationary time series are time dependent and the series do not have long-term equilibrium level.

Weakly stationary (covariance-stationary)

Time series is said to be weakly stationary if both the expected value of y_t and autocovariance of y_t are independent of time, i.e. E(y_t) = constant, var(y_t) = constant and the autocovariance cov(y_t, y_{t-s}) depends only on a lag s. This means that a time series does not have a trend, i.e. it has a finite mean, which is independent of time.

Difference-stationary

Time series is said to be difference-stationary if y_t is not stationary but $\Delta y_t = y_t - y_{t-1}$ is stationary. It is typical for a difference-stationary time series to "wander around" in time without a longer-term mean to revert to. Typical example is so called Random Walk (RW) process

$$y_t = y_{t-1} + u_t \tag{1.0}$$

where u_t is the error term.

On the other hand, random walk with a drift

$$y_t = \mu + y_{t-1} + u_t \text{, where} \tag{1.1}$$

μ is drift (constant).

Both the expected value and variance of y are time-dependent.

3.2 White Noise (unpredictable noise)

Weak white noise

Requirements for a weak white noise are the following: Firstly, the expected value of regression error term is zero. Secondly, the variance of the error term is constant (finite). Thirdly, the covariance between the error term at time t and at time t-s is zero. The interpretation of weak white noise is that the level is unpredictable.

There are several ways to test the white noise.

Simple test

$$|r_k| < 2 / \sqrt{T} \tag{1.2}$$

Box-Pierce

$$BP = T\sum_{1}^{K} r_K^2 \sim \chi^2(K) \tag{1.3}$$

Ljung-Box

$$LBQ = T(T+2) \sum_{1}^{K} \frac{r_K^2}{T-K} \sim \chi^2 \tag{1.4}$$

With monthly data, K = 12 is sensible. T = number of observations, r_K = autocorrelation.

If the error term is not white noise (WN), then one must always test the stationarity.

3.3 Unit root tests and hypothesis

Dickey-Fuller test (DF-test)

D.A. Dickey and W.A. Fuller (1979) have developed a method to test the existence of unit root. In a simple AR(1) model

$$y_t = \phi y_{t-1} + u_t \tag{1.5}$$

The hypothesis are the following:

$$H_0: \phi = 1 \text{ (nonstationary)}$$
$$H_1: \phi < 1 \text{ (stationary)}$$

This means that a unit root is present if $\phi = 1$. The model would be nonstationary in this case. If $\phi < 1$, then there is no unit root and the model is said to be stationary.

The regression model can be written as

$$\Delta y_t = \wp y_{t-1} + u_t \tag{1.6}$$

This model can be estimated and testing for a unit root is equivalent to testing $\wp = 0$. Since the test is done over the residual term rather than raw data, it is not possible to use standard t-distribution to as critical values. Therefore this statistic has a specific distribution known as the Dickey-Fuller table.

DF-test

$$\Delta y_t = \wp y_{t-1} + u_t \tag{1.7}$$

$$H_0: \wp = 0$$
$$H_1: \wp < 0$$

If the time series prove to be nonstationary, one has to differentiate it to come up with a stationary time series.

It has been shown that if there is a structural break in a time series the unit root tests are biased. This means that the unit root tests accept the presence of a unit root even if there is no unit root in reality.

3.4 Cointegration (common trend modelling)

Cointegration causes variables to move together in the long run (data are cointegrated because they respond to shocks together), which produces stationarity of the cointegrating combination despite the nonstationarity of its components. The long-run co-movements of variables in cointegrating system is ultimately driven by their dependence on underlying common stochastic trends (in this case x_t).Variables are said to be cointegrated if there is a linear combination of integrated variables that is stationary, that is, two or more nonstationary time series are cointegrated if a linear combination of these variables is stationary (converges to an equilibrium over time)

$x_t = x_{t-1} + e_{xt}$	I(1)	e_{xt} white noise
$y_t = y_{t-1} + e_{yt}$	I(1)	e_{yt} white noise
$y_t = \beta_0 + \beta_1 x_t + e_t$	I(0)	

where β_0 and β_1 are the cointegration parameters. All the variables should be integrated of the same order. But this does not mean that all the integrated variables are cointegrated. This means that there is no long-term equilibrium relation between the variables and that the variables can move in time randomly and independently of each other. If the variables are not integrated of the same order, one can conclude that the time series are not cointegrated.

3.5 Autoregressive processes

Many time series are processes, whose behaviour depends on previous observations, i.e. the lagged values of its own history. Yule's pth-order autoregression satisfies

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t \text{, where} \tag{1.8}$$

ϕ_0 is a constant term

ϕ_i is a parameter

$\varepsilon_t \sim NID(0,\sigma^2)$ (white noise) and $\{ y_t \}$ is a stationary process

3.6 Error correction model (ECM, Engle-Granger two-phase method)

Engle and Granger have presented the following method for an error correction model. Firstly, we test the stationarity of time series (x_t, y_t). Secondly, we estimate the level model and calculate the residuals. Thirdly, we estimate the ECM

$$\Delta y_t = \beta_0 + \gamma \hat{\varepsilon}_{t-1} \qquad \gamma < 0 \text{ and/or}$$
$$\Delta x_t = \delta_0 + \delta_1 \hat{\varepsilon}_{t-1} \qquad \delta > 0$$

If γ or δ is statistically significant, the model is said to be ECM. That means that the level model like AR- or ARMA model is also correct. The level model is for example

$$y_t = \alpha_0 + \alpha_1 x_t + \varepsilon_t$$

If there is a residual term, then y_t converges into x_t or vice versa or both.

3.7 AR-, MA- and ARMA modelling

In AR(p) (autoregressive) models, the lagged values of the dependent variable are the explanatory variables. In MA(q) (moving average) models, the explanatory items consist of the previous estimation errors. ARMA(p,q) (autoregressive moving average) models have both features.

The notation AR(p) refers to the autoregressive model of order p

$$y_t = \phi_0 + \sum_{i=1}^{p} \phi_i y_{t-i} + \varepsilon_t \tag{1.9}$$

In other words, AR(1) process is such where the current observation is explained by one lagged observation and an error term. The error term is white noise.

The notation MA(q) refers to the moving average model of order q

$$y_t = \varepsilon_t + \sum_{i=1}^{q} \theta \varepsilon_{t-i} \tag{2.0}$$

The notation ARMA(p,q) refers to the model with p autoregressive terms and q moving average terms

$$y_t = \sum_{i=1}^{p} \phi_i y_{t-i} + \sum_{i=1}^{q} \theta \varepsilon_{t-i} + \varepsilon_t \tag{2.1}$$

3.8 Heteroskedasticity (ARCH, GARCH, time-varying volatility modelling)

ARCH stands for auto-regressive conditional heteroskedasticity. Conditional variance means that the variance depends on time. Conditional heteroskedasticity means that series, whose conditional variance follows AR(1) process, has a constant long-term variance, but the variance at time t depends on the variance at time t-1.

An ARCH(q) process can be estimated using ordinary least squares (OLS). Here q stands for the lag length of ARCH error terms.

The level of the series is unpredictable but the variance is predictable. A simple model

$$y_t = \mu + u_t \tag{2.2}$$

The variance var(u_t) = h_t depends on time

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{2.3}$$

If $\alpha_1 = 0$, that means that the conditional variance is constant, $h_t = \alpha_0$ and there is no heteroskedasticity. The simplest way to test heteroskedasticity is to investigate, is there any autocorrelation between the squared OLS-residuals. That can be done for example with LBQ, Ljung-Box test.

On the other hand, if an ARMA model is assumed for the error variance, the model is a generalized auto-regressive conditional heteroskedasticity (GARCH) model. In that case, the GARCH(p,q) model is given by

$$\delta_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{i=1}^{p} \beta_i \delta_{t-i}^2 \tag{2.4}$$

where p is the order of the GARCH terms δ_t^2 and q is the order of ARCH terms u_t^2. It is worth to mention that all the parameters have to be positive. Furthermore, in order to come up with stable model, the sum of variables α_i and β_i have to be less than one. Otherwise, the estimated mean of the variance will be negative.

3.9 Estimation algorithm

The estimation algorithm goes in the following fashion: First, we estimate μ and calculate ut. Secondly, we estimate the model $u_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2$. As a result, we get $\hat{h}_t = \hat{\alpha}_0 + \hat{\alpha}_1 u_{t-1}^2$, a time-dependent variance estimate. Thirdly, we estimate the level μ using OLS-, or alternatively, weighted OLS-method (WLS,GLS) with weights $w_t = \frac{1}{\hat{h}_t}$. After that, we calculate u_t.

4 BASIC DATA DESCRIPTION

This section provides the data description and walks through the model estimation procedure.

The data consists of three time series: 3-month Euribor offered rate, 3-month US Treasury bill rate (T-bill) and spot rate of US dollar to Euro exchange rate. The starting date of every time series is 31.12.1998 and the ending date 31.12.2007. We have excluded the non-trading days from the analysis. The Euribor data consists of 2304 daily observations, T-bill data consists of 2345 daily observations and the exchange rate date consists of 2348 daily observations. All the calculations are done from the logarithmic data. The data is provided by Bloomberg.

4.1 Euribor

The Euro Interbank Offered Rate (or Euribor) is a daily reference rate based on the averaged interest rates at which banks offer to lend unsecured funds to other banks in the euro interbank market.

Euribor	
Mean	3.209865017
Standard Error	0.019520398
Median	3.2065
Mode	2.116
Standard Deviation	0.9369791
Sample Variance	0.877929835
Kurtosis	-1.078044978
Skewness	0.398404747
Range	3.183
Minimum	1.957
Maximum	5.14
Sum	7395.529
Count	2304
Confidence Level(95.0%)	0.038279394

Figure 4.1.1 Descriptive statistics of Euribor

Figure 4.1.1 shows the descriptive statistics of Euribor. The maximum value of Euribor was 5.14%, minimum being 1.96% and mean 3.21%, respectively.

Figure 4.1.2 Euribor rates from 31.12.1998 – 31.12.2007

Euribor rose quite sharply at the beginning and reached its peak on 31st October 2000. Then it started to decrease and maintained historically low level at a bit more than 2% for a quite long time in 2004 – 2005. After that it has risen steadily again.

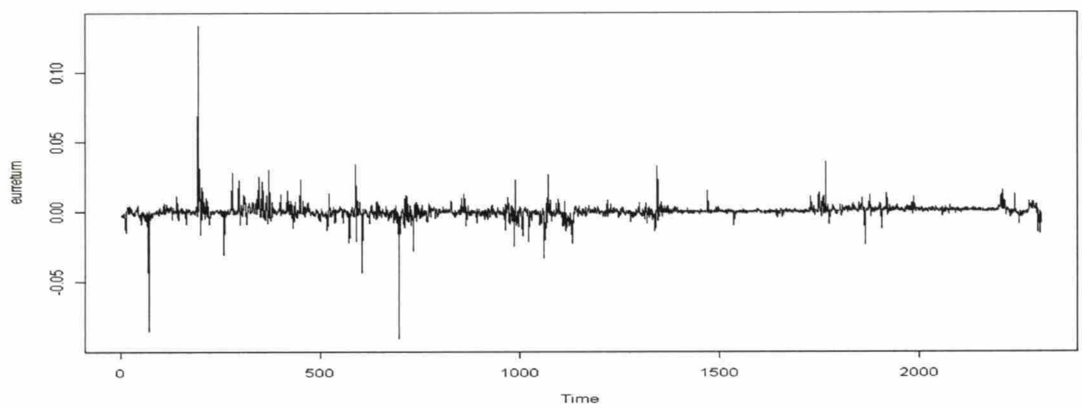

Figure 4.1.3 Euribor returns from 31.12.1998 – 31.12.2007

Figure 4.1.3 shows the return series of Euribor. As can be seen, the returns are concentrated around zero. This is in line with the stationarity hypothesis.

4.2 US T-Bill

Treasury bills (or T-bills) mature in one year or less. Like zero-coupon bonds, they do not pay interest prior to maturity; instead they are sold at a discount of the par value to create a positive yield to maturity. Treasury bills are regarded as the least risky investment available to U.S. investors. Banks and financial institutions, especially primary dealers, are the largest purchasers of T-Bills. T-Bills are quoted for purchase and sale in the secondary market on an annualized percentage yield to maturity.

T-Bill	
Mean	3.408547079
Standard Error	0.035872024
Median	3.626
Mode	0.9273
Standard Deviation	1.737109955
Sample Variance	3.017550997
Kurtosis	-1.424819326
Skewness	-0.073511448
Range	5.6302
Minimum	0.8048
Maximum	6.435
Sum	7993.0429
Count	2345
Confidence Level(95.0%)	0.070344197

Figure 4.2.1 Descriptive statistics of T-Bill

Figure 4.2.1 shows the descriptive statistics of T-Bill. The maximum value of T-Bill was 6.44%, minimum being 0.80% and mean 3.41%, respectively.

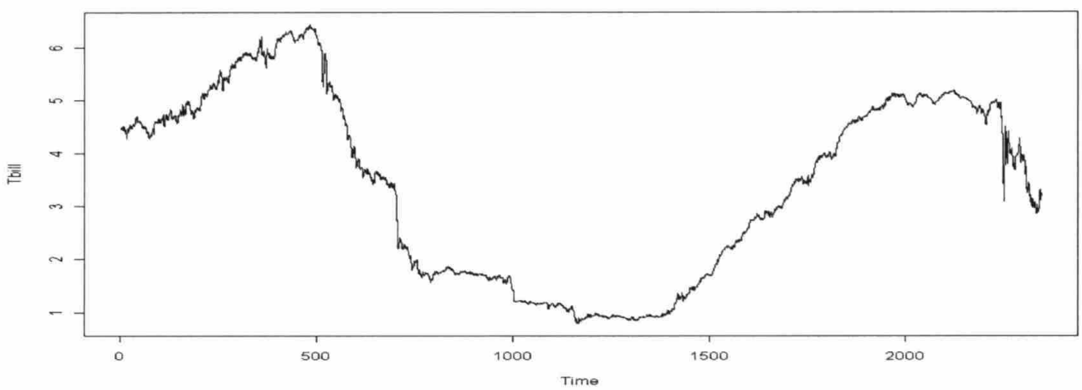

Figure 4.2.2 T-Bill yields from 31.12.1998 – 31.12.2007

As can be seen in the Figure 4.2.2, the T-Bill yields followed a similar kind of pattern than three month Euribor rates.

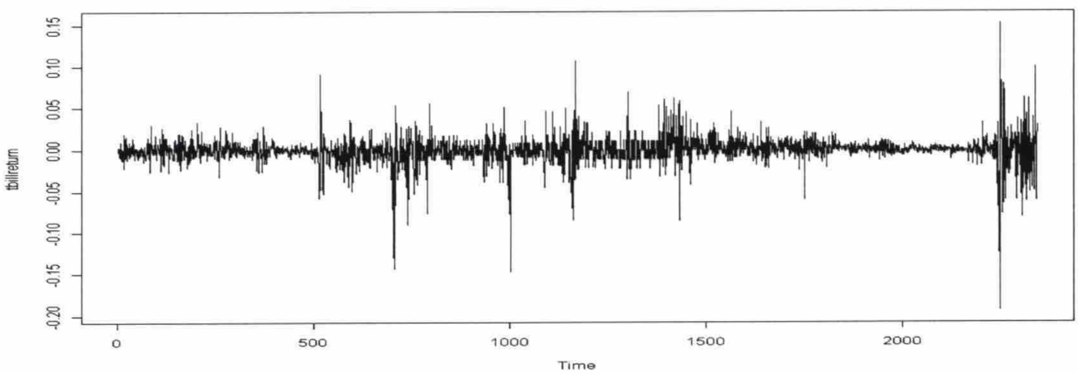

Figure 4.2.3 US T-Bill returns from 31.12.1998 – 31.12.2007

Figure 4.2.3 shows the return series of T-Bill. The figure indicates that T-Bill has been more volatile than Euribor.

4.3 Euro-US dollar exchange rate

The euro was introduced to world financial markets as an accounting currency in 1999 and launched as physical coins and banknotes at the beginning of 2002. It replaced the former European Currency Unit (ECU) at a ratio of 1:1.

Euro-US dollar exchange rate	
Mean	1.11991431
Standard Error	0.003464681
Median	1.1476
Mode	1.2629
Standard Deviation	0.167885173
Sample Variance	0.028185431
Kurtosis	-1.189683338
Skewness	-0.025355883
Range	0.6603
Minimum	0.8255
Maximum	1.4858
Sum	2629.5588
Count	2348
Confidence Level(95.0%)	0.006794154

Figure 4.3.1 Descriptive statistics of Euro-US dollar exchange rate

Figure 4.3.1 shows the descriptive statistics of Euro-US dollar exchange rate. The maximum value of Euro-US dollar exchange rate was 1.49, minimum being 0.83 and mean 1.12, respectively.

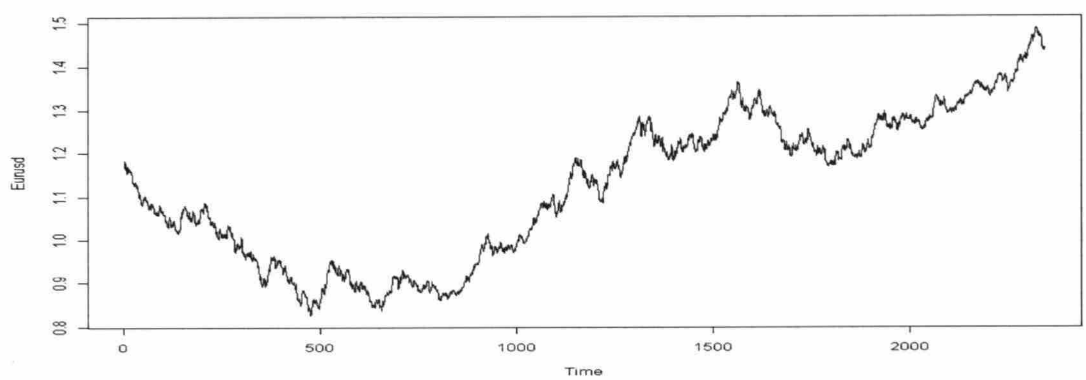

Figure 4.3.2 Euro-US dollar exchange rates from 31.12.1998 – 31.12.2007

The researched exchange change time series could be divided into two parts. Firstly, US dollar appreciated against Euro. After the introduction of the euro, its exchange rate against other currencies fell heavily, especially against the U.S. dollar. At its introduction in 1999, the euro was traded at US$1.18/€, but by October 26, 2000, it had fallen to an all-time low of $0.83/€.

After the appearance of the coins and notes in January 1st, 2002 and the replacement of all national currencies, the euro then began steadily appreciating, and soon regained parity with the U.S. dollar.

Since December 2002, the euro has not again fallen below parity with the U.S. dollar but has, instead, begun an unprecedented appreciation.

On 23 May 2003, the euro surpassed its initial ($1.18) trading value for the first time. At the end of 2004, it reached a peak of $1.3668 as the U.S. dollar fell against all major currencies, fuelled by the so-called double deficit in the US accounts. The dollar temporarily recovered in 2005, rising to $1.18 in July 2005, and was stable throughout the second half of 2005. The steep increase in U.S. interest rates during 2005 had much to do with this trend. But on November 2005 the dollar again began to fall steadily until the present day hitting one record low after another. On 23 April 2008, the U.S. dollar fell to an all-time low of $1.5940 against the euro.

Euro is the single currency for over 320 million Europeans. Including areas using currencies pegged to the euro, the euro directly affects close to 500 million people worldwide. With more than €610 billion in circulation as of December 2006 (equivalent to US$802 billion at the exchange rates at the time), the euro is the currency with the highest combined value of cash in circulation in the world, having surpassed the U.S. dollar.

Taking the official estimates of 2007 GDP, the Eurozone is the largest economy in the world by March 2008 after the €/$ exchange rate surpassed 1.56. (source: European Central Bank)

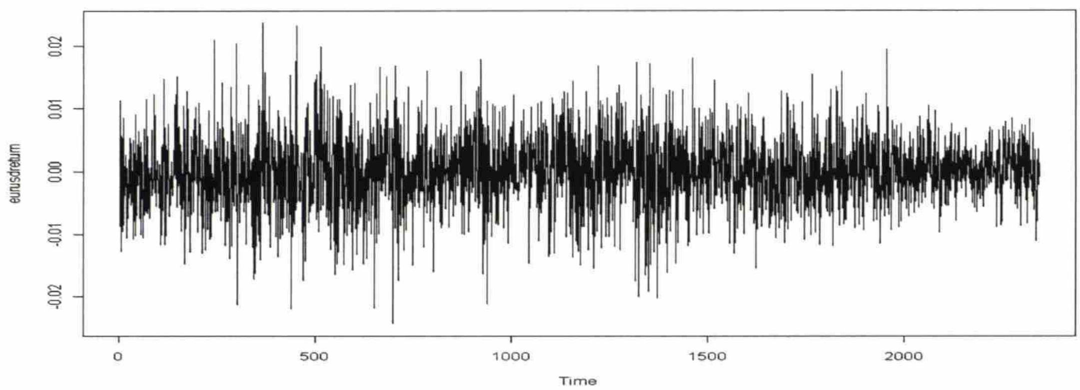

Figure 4.3.3 Euro-US dollar exchange rate returns from 31.12.1998 – 31.12.2007

Figure 4.3.3 shows the return series of Euro-US dollar exchange rate. The returns are quite evenly distributed between -1% and 1%.

4.4 Model

OLS-regression is estimated for each interest rate and exchange rate series in order to get equilibrium levels.

$$\Delta Y = a + bY_{t-1} + c_1\Delta Y_{t-1} + \ldots + c_k\Delta Y_{t-k} + u_t, \text{ where} \tag{3.0}$$

Y is yield at time t, Δ stands for changes, u_t is a sequence of independent, normally distributed random variables with mean zero and constant variance (white noise), k is the number of lagged changes.

We picked up the error-terms from the estimations and use them to estimate the future change in interest rate series, i.e. volatility.

5 METHODS, TESTS AND RESULTS

In this section we present the results of the tests we did and provide the outcome of the models.

All the tests were done by using the econometrics software called R. In the regressions we used 95% confidence level.

5.1 Unit root tests (test of stationarity)

The critical values for Dickey-Fuller test with a sample size more than 750 (infinity) is -3.434 with p-value 0.01 (99% confidence level) and -2.862 with p-value 0.05 (95% confidence level). The Dickey-Fuller values are always negative. The larger the absolute value of Dickey-Fuller statistic is compared to critical value, the stronger we reject the unit root hypothesis. There is also an alternative unit root test called the Phillips-Perron test. The corresponding critical values are the same.

Dickey-Fuller and Phillips-Perron tests (table 1 in Appendix 1) indicate that all the time series I tested are non-stationary. That means that we have to differentiate the time series, i.e. investigate the returns. Unit root tests of the returns (table 2 in Appendix 2) show that the return time series are stationary. This indicates that the original time series were integrated of order one, i.e. they had exactly one unit root.

Testing a unit root is closely related to testing whether two time series are cointegreated, i.e. do they have a common trend. If, for example, an exchange rate time series were cointegrated with another exchange rate time series that would mean that one could use the other time series for forecasting the behaviour of the other. That would be a violation of the efficient market hypothesis. But on the other hand, it is reasonable to expect that for example Euro - US dollar spot rate time series is cointegrated with the Euro – US dollar forward rate time series.

5.2 Autocorrelation tests

The feature of autocorrelation (MA-part in ARMA model) and partial autocorrelation (AR-part in ARMA model) can be studied through graphs. In the graphs, you can see the number of significant lags with 95% confidence level, vertical line indicating the significance level.

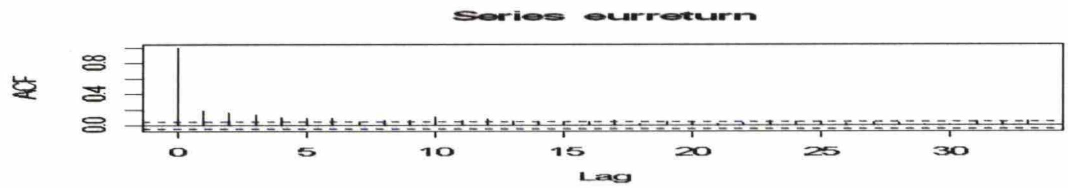

Figure 5.2.1. Autocorrelation function of Euribor returns.

Figure 5.2.1 shows that there are as much as six significant MA lags in the Euribor returns series.

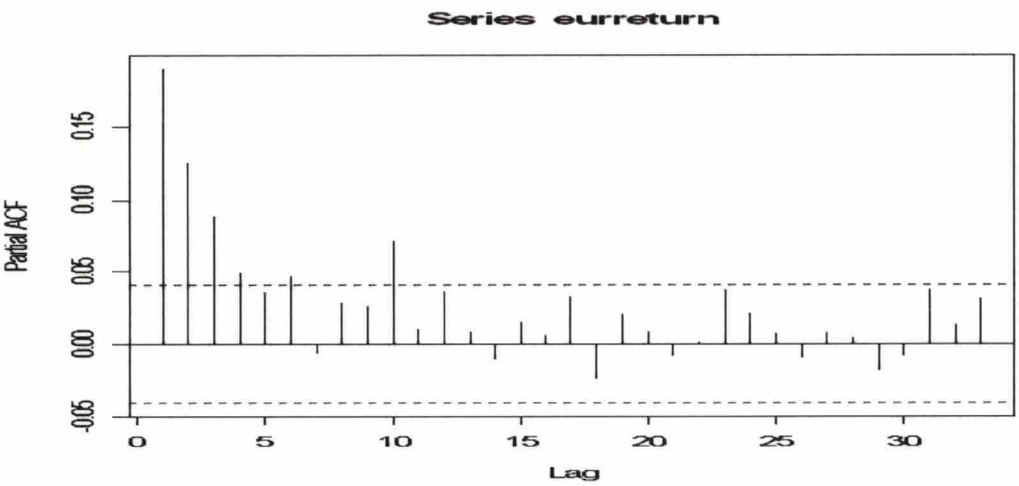

Figure 5.2.2 Partial autocorrelation function of Euribor returns.

Figure 5.2.2 indicates that there could be four significant AR lags in the Euribor returns series.

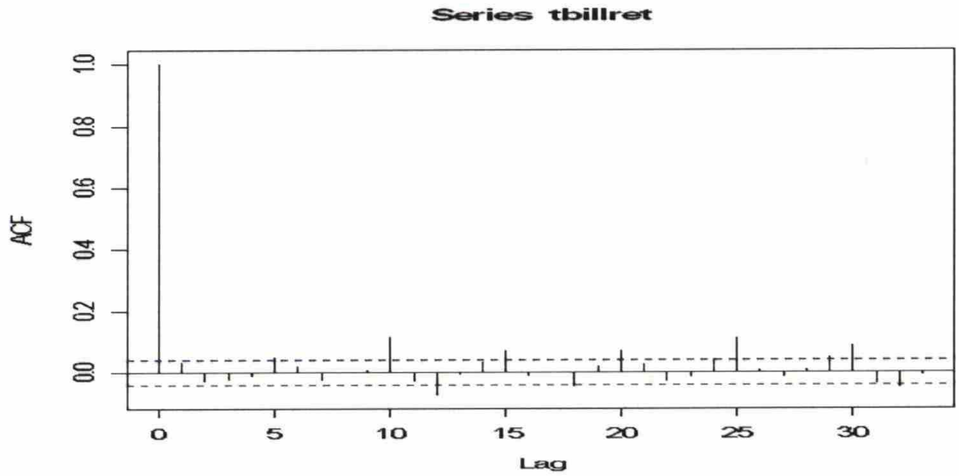

Figure 5.2.3 Autocorrelation function of T-Bill returns

The following figures from 5.2.3 to 5.2.6 illustrate the corresponding significant AR and MA lags for T-Bill and Euro-US dollar exchange rate returns series, respectively. As can be seen, the figures seem not to prove any autocorrelation between the lags. That is why we have to take a closer look at the AR and MA lags by estimating the regression functions.

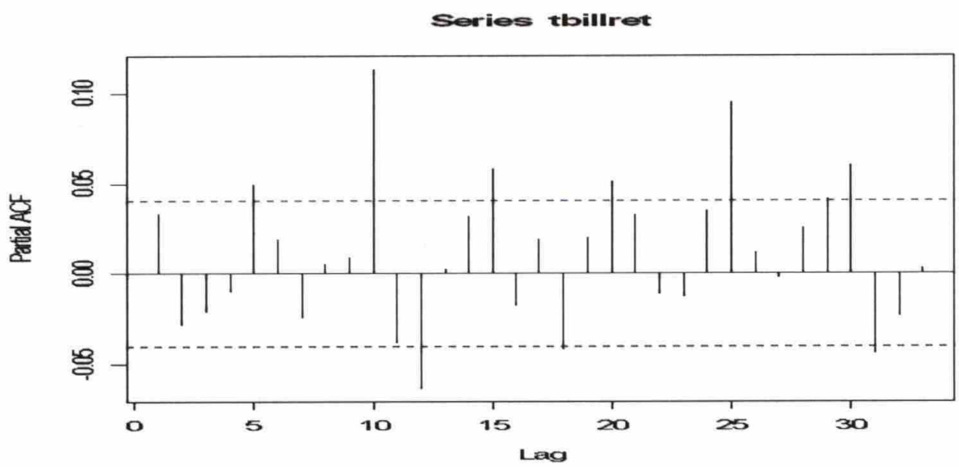

Figure 5.2.4 Partial autocorrelation function of T-Bill returns.

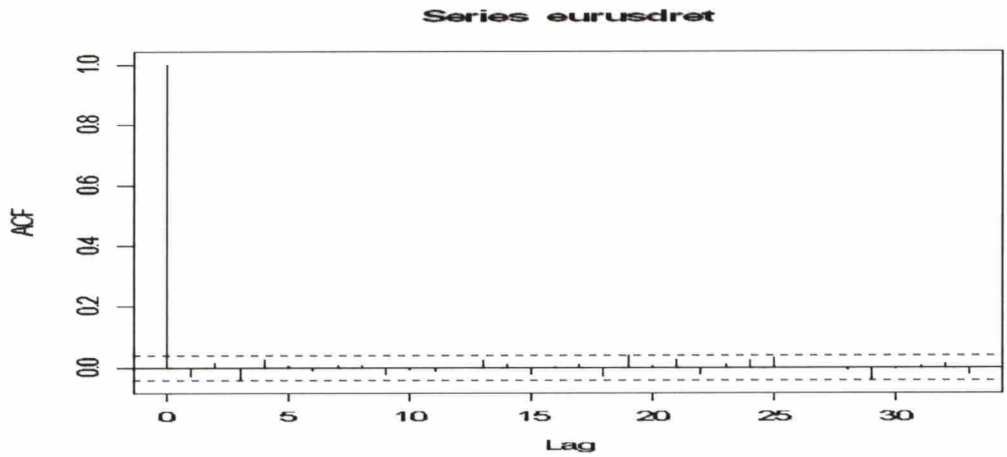

Figure 5.2.5 Autocorrelation function of Euro-US dollar exchange rate returns.

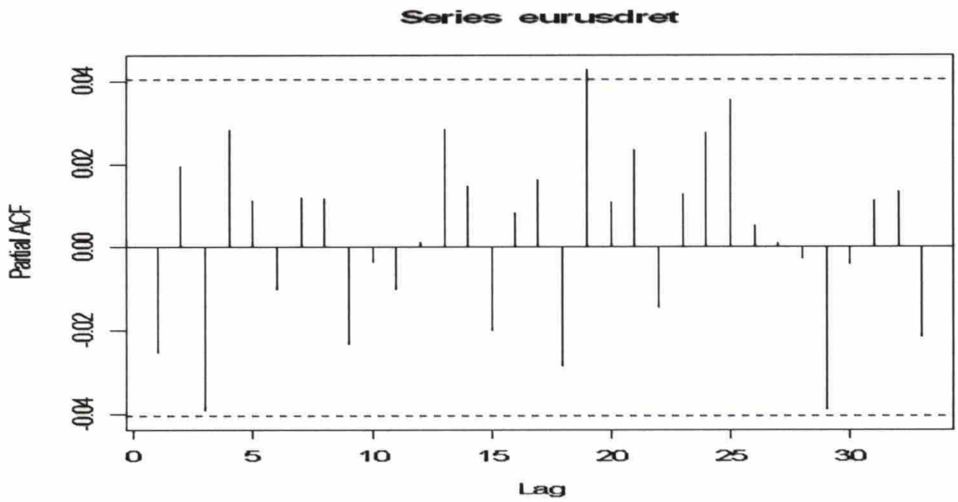

Figure 5.2.6 Partial autocorrelation function of Euro-US dollar exchange rate.

5.3 ARMA modelling

We found that the most suitable model ARMA model (Table 3 in Appendix 3) for Euribor is

ARMA(2,2)

$$y_t = 1.735\, y_{t-1} - 0.7379\, y_{t-2} - 1.599\, \varepsilon_{t-1} + 0.609\, \varepsilon_{t-2} \qquad (4.0)$$

Among all the plausible models we choose ARMA(2,2) because it has the lowest AIC value (Akaike information criterion). In general case

$$AIC = 2k - 2\ln(L) \text{ ,where} \tag{4.1}$$

k is the number of parameters in the model
L is the maximized value of the likelihood function for the estimated model.

If it is assumed that the model errors are normally and independently distributed. Then AIC becomes

$$AIC = 2k + n[\ln(2\pi RSS/n) + 1] \text{ ,where} \tag{4.2}$$

n is the number of observations
RSS is the residual sum of squares

$$RSS = \sum_{i=1}^{n} \hat{\varepsilon}_i^2 \tag{4.3}$$

Increasing the number of free parameters to be estimated improves the goodness of fit. Hence AIC not only rewards goodness of fit but also includes a penalty that is in increasing function of the number of estimated parameters. This penalty discourages overfitting. That is why the preferred model is the one with lowest AIC value. The AIC methodology attempts to find the model that best explains the data with a minimum of free parameters.

Also, by using the same criterion, the most suitable ARMA model for the USD-Euro exchange rate (Table 4 in Appendix 3) is

ARMA(2,3)

$$y_t = -0.9371\, y_{t-1} - 0.924\, y_{t-2} + 0.9239\, \varepsilon_{t-1} + 0.9187\, \varepsilon_{t-2} - 0.03849\, \varepsilon_{t-3} \tag{4.4}$$

Furthermore, the most suitable ARMA model for T-Bill (Table 5 in Appendix 3) is

ARMA(2,2)

$$y_t = 0.6184\, y_{t-1} - 0.9971\, y_{t-2} - 0.6286\, \varepsilon_{t-1} + 0.9836\, \varepsilon_{t-2} \tag{4.5}$$

5.4 Tests of normality of residuals

The normality can be tested by examining the skewness and kurtosis of the distribution. If the skewness and kurtosis test values are close to zero then the series is likely to follow a normal distribution.

The normality of the residual terms was tested with Jarque-Bera and Shapiro-Wilk normality tests. The critical value for Shapiro-Wilk test is 0.947 with 95% significance level. The W values is always smaller or equal to one. If the W value is too small compared to critical value then the null hypothesis, that the residuals are from normal distribution, is rejected.

When we tested for the normality of the residual terms (Table 6 in Appendix 4) yielded by the plausible ARMA models, we found that we have to reject the null hypothesis that the residuals are normally distributed for Euribor and T-Bill but for Euro-US dollar exchange rate residuals are normal.

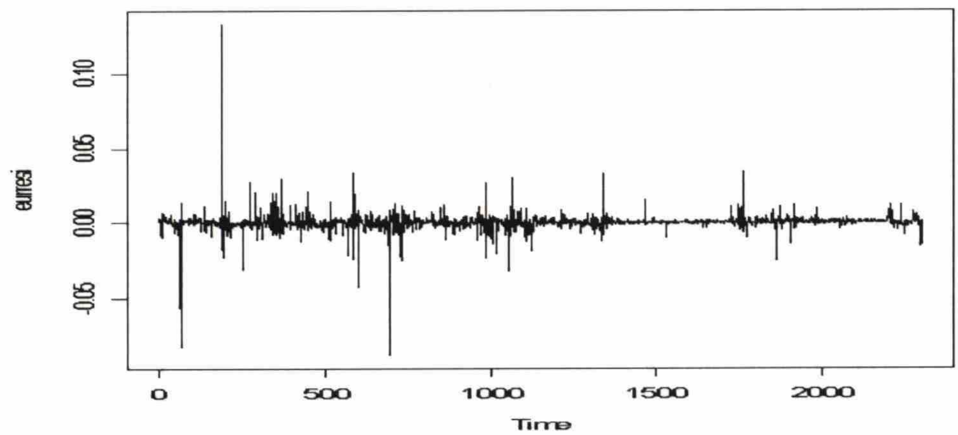

Figure 5.4.1 The residuals from Euribor ARMA(2,2) model.

Figure 16 shows that the Euribor residual terms are quite evenly distributed around zero. This might speak for the behalf of normal distribution of the residual terms.

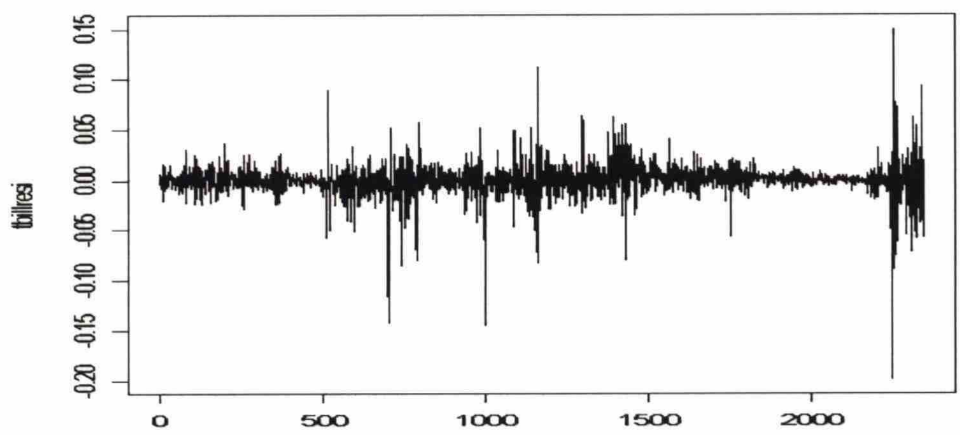

Figure 5.4.2 The residual terms of T-Bill ARMA(2,2) model.

Figure 17 shows that there are much more volatility in the T-Bill residual terms that in the corresponding Euribor residuals.

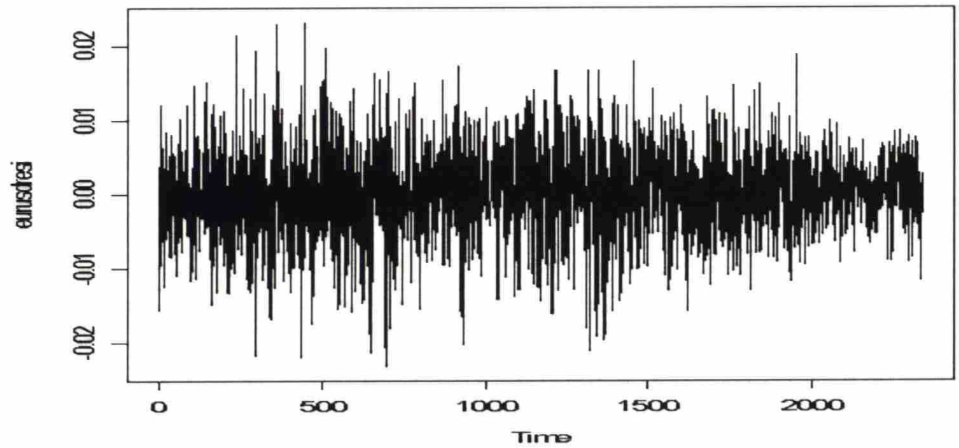

Figure 5.4.3 The residual terms of Euro-US dollar exchange rate ARMA(2,3) model.

Figure 18 shows that the exchange rate residual terms are very tightly distributed around zero. This is in line with the results of the normality tests, which indicate that the exchange rate residuals are normally distributed.

5.5 GARCH modelling

If the error terms follow a (G)ARCH process, the squared residuals should exhibit an AR(MA) structure. This means that if there are heteroskedasticity in the residual terms then ARMA is a suitable level model. On the other hand, if there is no heteroskedasticity in the residual, then AR or MA model could be more suitable as a level model. The residual are tested for heteroskedasticity by Ljung-Box test. GARCH model is applied if the residuals prove to be heteroskedastic.

The critical chi-squared value with 5% significance level and degrees of freedom = 1 is 3.84. If the tested chi-squared value is greater than the critical value then the null hypothesis of no-autocorrelation is rejected.

I investigated the most suitable GARCH models for Euribor (Table 7 in Appendix 5).

All the plausible GARCH models for Euribor fail to prove the existence of autocorrelation between the squared residuals. AR- or MA model could be seen as more suitable for Euribor.

I founded that the most suitable GARCH model for exchange rate (Table 8 in Appendix 5) is

GARCH(3,2)

$$\delta_t^2 = 0.0000112 + 0.1543\,u_{t-2}^2 + 0.353\,\delta_{t-1}^2 \qquad (4.6)$$

Here u_t^2 stands for squared residuals. This model is valid with 10% significance level and u_{t-2}^2 and δ_{t-1}^2 are the only significant lags yielded by this GARCH(3,2) model.

According to my tests the most suitable GARCH model for T-Bill (Table 9 in Appendix 5) is

GARCH(2,3)

$$\delta_t^2 = 0.00000167 + 0.117\,u_{t-1}^2 + 0.133\,u_{t-2}^2 + 0.0929\,u_{t-3}^2 + 0.713\,\delta_{t-2}^2 \qquad (4.7)$$

Here δ^2 is the variance of the lagged estimate. However, this model proves not to be stable since the sum of the parameters is greater than one.

5.6 Cointegration test

We tested the cointegration of the Euribor and T-Bill by using the Phillips-Ouliaris cointegration test. The critical value is 3.37 with 95% significance level and -3.96 with 99% significance level. If the test value is smaller than the critical value then the null hypothesis of no cointegration is rejected.

According to our test (Table 10 in Appendix 6) the Euribor and T-Bill time series are cointegrated. This means that these two time series have a common trend and one could use the other time series for forecasting the other.

6 FORECASTING

In this section we provide the historical volatilities of the examined time series as well as the results of the volatility forecasting of Euro-US dollar exchange rate.

6.1 Description of historical volatilities

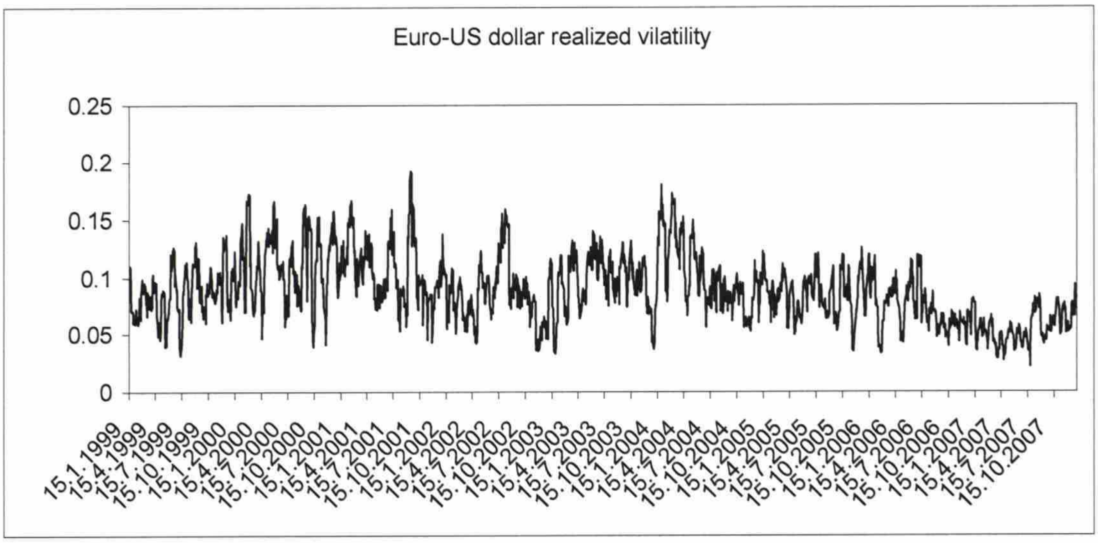

Figure 6.1.1 10-day moving average volatility of Euro-US dollar exchange rate between Jan 1999 and Dec 2007.

Figure 6.1.1 shows the historical volatility of Euro-US dollar exchange rate. It has ranged between 5 % and 15%.

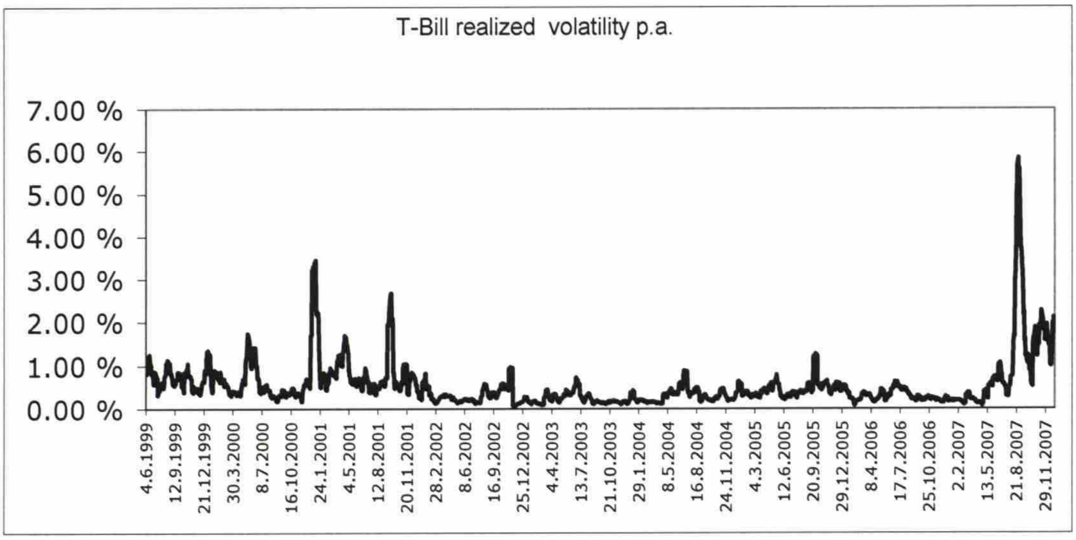

Figure 6.1.2 10-day moving average volatility of T-Bill yield between Jan 1999 and Dec 2007

Figure 6.1.2 shows that the T-Bill volatility has been very low historically. Typically it has been under 1% but very recently it has risen quite sharply.

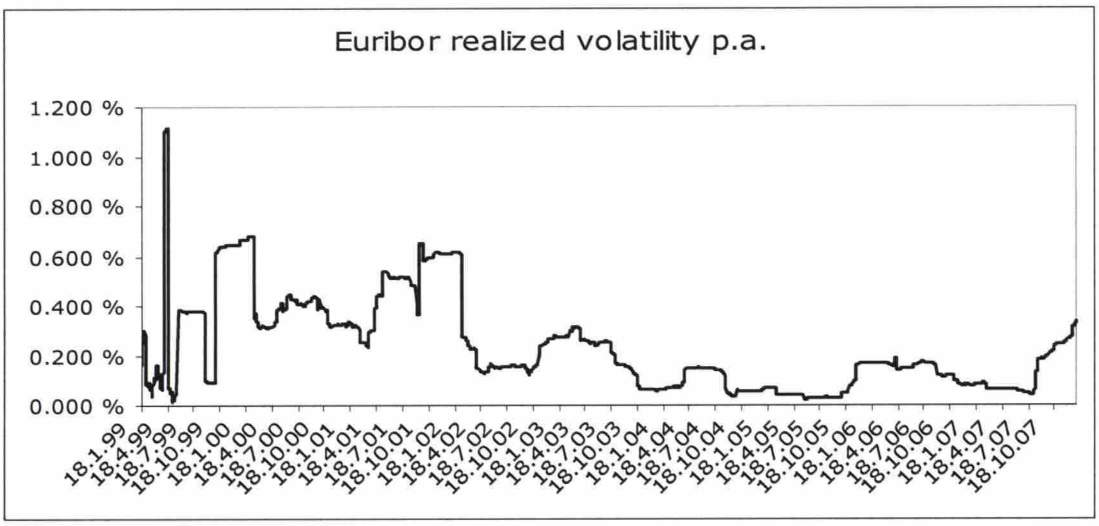

Figure 6.1.3 10-day moving average volatility of 3 month Euribor yield between Jan 1999 and Dec 2007

Figure 6.1.3 shows that, like T-Bill, 3 month Euribor volatility is very low historically as well. Also Euribor volatility has experienced quite rapid rise in the recent history.

6.2 Euro-US dollar volatility forecasting

We used a time period from 28th August 2007 to 31st December 2007 to calculate a 90-day moving average volatility time series for Euro - US dollar exchange rate time series. The one-step-ahead daily volatility forecasts are produced by the GARCH(3,2) model. The procedure is repeated 90 times in order to produce 90 daily volatility forecasts for evaluation. Hence, the last forecast is for 31st December 2007.

We compute the following regression for the assessment of the predictability of our models

$$\delta_{realized,t+1} = \alpha + \beta\hat{\delta}_{forecast,t+1} + \varepsilon_t \tag{4.8}$$

where $\delta_{realized,t+1}$ is the realized volatility at time t+1 and $\hat{\delta}_{forecast,t+1}$ is the forecasted value for true volatility of time t+1 predicted at time t.

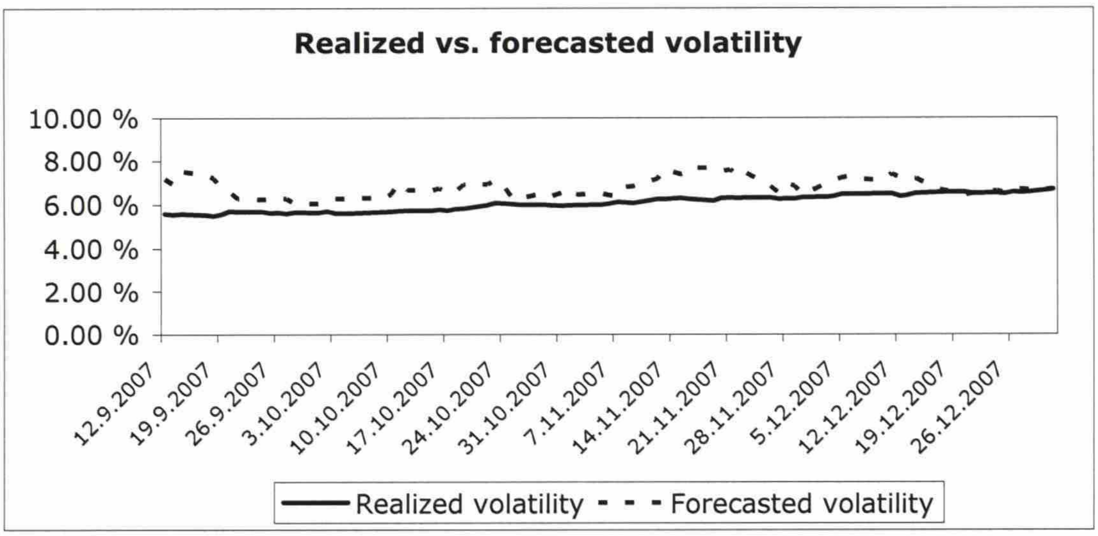

Figure 6.2.1 The realized and forecasted volatilities of Euro-US dollar exchange rate between Aug 2007 and Dec 2007.

Figure 6.2.1 shows the comparison of the realized volatility of Euro-US dollar exchange rate and the forecast yielded by the GARCH(3,2) model

$$\delta_t^2 = 0.0000112 + 0.1543\,u_{t-2}^2 + 0.353\,\delta_{t-1}^2 \tag{4.9}$$

As can be seen in the figure, the forecast seems to be too high and too fluctuating compared to the realized volatility. The estimated long term mean for the variance V was calculated to be 7.54% p.a., which is pretty much in line with the realized values. The estimated variance mean was given by formulas

$$\gamma = 1 - \alpha - \beta \tag{5.0}$$

and

$$V = \frac{\omega}{\gamma} \tag{5.1}$$

where α = 0.1543, β = 0.353 and ω = 0.0000112. This daily estimate is then transformed to per annum volatility.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.345934
R Squared	0.11967
Adjusted R Squared	0.108237
Standard Error	0.004056
Observations	79

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.000172	0.000172	10.46720945	0.001791799
Residual	77	0.001267	1.65E-05		
Total	78	0.001439			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.042158	0.007952	5.301498	1.06412E-06
X Variable 1	0.423064	0.130765	3.235307	0.001791799

Figure 6.2.2 The summary output of forecast regression.

The regression variable was found to be statistically significant at 99% confidence level. Furthermore, the model's explanatory power measured by R^2 was about 12%. Hence, we can say that the model yields somewhat reasonable forecasts for the volatility.

7 CONCLUSIONS

In this thesis we have examined time series analysis and volatility modelling from the interest rate and exchange rate point of view. Literature review covered the recent developments in the field of GARCH modelling and highlighted the research results of interest rate and exchange rate modelling. We employed error correction procedure to come up with suitable autoregressive and conditional heteroskedasticity models for volatility modelling.

Based on the results of our study we found that the three month US T-Bill interest rate, three month Euribor interest rate and US dollar-Euro exchange rate time series are non-stationary, i.e. the time series are said to have a unit root. We differentiated the time series once, meaning that we examined the return time series. All those time series proved to be stationary. That means that the original time series had exactly one unit root, i.e. they were integrated of order one.

We studied the autocorrelations of the return time series and fitted an OLS-regression to come up with suitable level model. These level models are typically autoregressive processes (AR models), MA processes (MA models) or then they have both AR and MA features (ARMA models). We found that the most suitable level model for Euribor is ARMA(2,2). When we examined the residual terms they proved not to feature heteroskedasticity, so GARCH model is was not applied. The most suitable ARMA model for US T-Bill was found to be ARMA(2,2) and the corresponding GARCH model is GARCH(2,3). However, that particular model proves not to be very stable so it was not used in forecasting. Furthermore, we found that the most suitable level model for USD-Euro exchange rate is ARMA(2,3) and the GARCH model GARCH(3,2). The forecasts for the Euro-US dollar exchange rate volatility were found to be somewhat reasonable.

Due to the importance of both interest rate and currency markets for the economy volatility models are being developed further and further. For a future research, it would be interesting to examine whether the interest rate differential between Euro area and United States can explain the behaviour of Euro-US dollar exchange rate.

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APPENDICES

APPENDIX 1

TABLES AND TESTS

Table 1.

Unit root test of Euribor

H_0: non-stationary
H_1: stationary

Phillips-Perron Unit Root Test

data: Euribor
Dickey-Fuller = 0.4782, Truncation lag parameter = 8, p-value = 0.99

Unit root test of Eur-USD exchange rate

Phillips-Perron Unit Root Test

data: Eurusd
Dickey-Fuller = -2.8908, Truncation lag parameter = 8, p-value = 0.2012

Unit root test of US T-Bill

Phillips-Perron Unit Root Test

data: Tbill
Dickey-Fuller = -0.7554, Truncation lag parameter = 8, p-value = 0.9656

APPENDIX 2

Table 2.

Analysis of Euribor returns

Phillips-Perron Unit Root Test

H_0: non-stationary
H_1:stationary

data: eurreturn
Dickey-Fuller = -41.6152, Truncation lag parameter = 8, p-value = 0.01

Analysis of Eur-USD exchange rate returns

Phillips-Perron Unit Root Test

data: eurusdret
Dickey-Fuller = -49.7052, Truncation lag parameter = 8, p-value = 0.01

Analysis of US T-Bill returns

Phillips-Perron Unit Root Test

data: tbillret
Dickey-Fuller = -46.7508, Truncation lag parameter = 8, p-value = 0.01

APPENDIX 3

Table 3. Plausible Euribor ARMA models

ARMA(1,0)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	0.1902603	0.0204511	9.303	<2e-16 ***
intercept	0.0001303	0.0001187	1.097	0.272

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 3.245e-05, Conditional Sum-of-Squares = 0.07, AIC = -17271.22

ARMA(0,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ma1	0.1544970	0.0186367	8.290	2.22e-16 ***
intercept	0.0001600	0.0001375	1.163	0.245

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 3.269e-05, Conditional Sum-of-Squares = 0.08, AIC = -17253.96

ARMA(1,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	8.794e-01	3.368e-02	26.113	<2e-16 ***
ma1	-7.527e-01	4.813e-02	-15.638	<2e-16 ***
intercept	1.981e-05	2.941e-05	0.674	0.5

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 3.143e-05, Conditional Sum-of-Squares = 0.07, AIC = -17342.99

ARMA(2,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)

ar1	9.707e-01	5.800e-02	16.735	<2e-16 ***
ar2	-4.815e-02	3.113e-02	-1.547	0.122
ma1	-8.272e-01	5.335e-02	-15.506	<2e-16 ***
intercept	1.334e-05	2.085e-05	0.640	0.522

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 3.141e-05, Conditional Sum-of-Squares = 0.07, AIC = -17342.33

ARMA(1,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	9.134e-01	3.185e-02	28.679	<2e-16 ***
ma1	-7.720e-01	3.889e-02	-19.852	<2e-16 ***
ma2	-3.709e-02	2.580e-02	-1.438	0.151
intercept	1.481e-05	2.290e-05	0.647	0.518

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 3.141e-05, Conditional Sum-of-Squares = 0.07, AIC = -17342.06

ARMA(2,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	1.735e+00	6.462e-02	26.848	<2e-16 ***
ar2	-7.379e-01	6.369e-02	-11.586	<2e-16 ***
ma1	-1.599e+00	7.371e-02	-21.692	<2e-16 ***
ma2	6.090e-01	7.088e-02	8.592	<2e-16 ***
intercept	9.969e-07	1.255e-06	0.794	0.427

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 3.129e-05, Conditional Sum-of-Squares = 0.07, AIC = -17349.25

Table 4. Plausible Exchange rate ARMA models

ARMA(1,0)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-0.0250603	0.0206370	-1.214	0.225
intercept	0.0000868	0.0001224	0.709	0.478

Fit:
sigma^2 estimated as 3.513e-05, Conditional Sum-of-Squares = 0.08, AIC = -17377.44

ARMA(0,1)
Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ma1	-2.442e-02	2.029e-02	-1.204	0.229
intercept	8.507e-05	1.194e-04	0.712	0.476

Fit:
sigma^2 estimated as 3.513e-05, Conditional Sum-of-Squares = 0.08, AIC = -17377.41

ARMA(1,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-0.7055549	0.1435219	-4.916	8.83e-07 ***
ma1	0.6745409	0.1483960	4.546	5.48e-06 ***
intercept	0.0001536	0.0002055	0.747	0.455

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Fit:
sigma^2 estimated as 3.511e-05, Conditional Sum-of-Squares = 0.08, AIC = -17377.28

ARMA(2,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-0.5915461	0.2424281	-2.440	0.0147 *
ar2	0.0185227	0.0227042	0.816	0.4146
ma1	0.5684440	0.2426856	2.342	0.0192 *
intercept	0.0001349	0.0001936	0.697	0.4859

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Fit:
sigma^2 estimated as 3.51e-05, Conditional Sum-of-Squares = 0.08, AIC = -17375.81

ARMA(1,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-0.6009011	0.2388402	-2.516	0.0119 *
ma1	0.5778438	0.2388800	2.419	0.0156 *
ma2	0.0186286	0.0234852	0.793	0.4277
intercept	0.0001377	0.0001969	0.699	0.4843

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 3.51e-05, Conditional Sum-of-Squares = 0.08, AIC = -17375.77

ARMA(2,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-0.5875794	0.2767664	-2.123	0.0338 *
ar2	0.1036915	0.2918192	0.355	0.7223
ma1	0.5631717	0.2801909	2.010	0.0444 *
ma2	-0.0901012	0.3000550	-0.300	0.7640
intercept	0.0001280	0.0001866	0.686	0.4928

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 3.51e-05, Conditional Sum-of-Squares = 0.08, AIC = -17373.94

ARMA(3,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-0.8277598	0.8968778	-0.923	0.356
ar2	-0.6982265	0.7521235	-0.928	0.353
ar3	-0.0489821	0.0319728	-1.532	0.126
ma1	0.8039030	0.8965670	0.897	0.370
ma2	0.6927631	0.7457378	0.929	0.353
intercept	0.0001983	0.0003114	0.637	0.524

Fit:
sigma^2 estimated as 3.505e-05, Conditional Sum-of-Squares = 0.08, AIC = -17375.18

ARMA(2,3)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-9.371e-01	1.776e-02	-52.762	<2e-16 ***
ar2	-9.240e-01	2.899e-02	-31.872	<2e-16 ***

ma1	9.239e-01	2.695e-02	34.286	<2e-16 ***
ma2	9.187e-01	3.264e-02	28.146	<2e-16 ***
ma3	-3.849e-02	2.114e-02	-1.820	0.0687 .
intercept	2.455e-05	3.411e-04	0.072	0.9426

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:

sigma^2 estimated as 3.500e-05, Conditional Sum-of-Squares = 0.08, AIC = -17378.16

ARMA(3,3)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-0.5455007	9.1474247	-0.060	0.952
ar2	-0.8863136	0.5603758	-1.582	0.114
ar3	-0.3658861	7.2623907	-0.050	0.960
ma1	0.5165889	9.1773683	0.056	0.955
ma2	0.8844900	0.5498604	1.609	0.108
ma3	0.3167390	7.2306708	0.044	0.965
intercept	0.0001920	0.0006158	0.312	0.755

Fit:

sigma^2 estimated as 3.500e-05, Conditional Sum-of-Squares = 0.08, AIC = -17376.17

Table 5. Plausible T-Bill ARMA models

ARMA(1,0)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	0.0335993	0.0206565	1.627	0.104
intercept	-0.0001318	0.0003445	-0.383	0.702

Fit:
sigma^2 estimated as 0.0002782, Conditional Sum-of-Squares = 0.65, AIC = -12534.52

ARMA(0,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ma1	0.0354486	0.0211911	1.673	0.0944 .
intercept	-0.0001372	0.0003566	-0.385	0.7005

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 0.0002782, Conditional Sum-of-Squares = 0.65, AIC = -12534.67

ARMA(1,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-0.4168068	0.3730709	-1.117	0.264
ma1	0.4528848	0.3658101	1.238	0.216
intercept	-0.0001930	0.0005028	-0.384	0.701

Fit:
sigma^2 estimated as 0.0002781, Conditional Sum-of-Squares = 0.65, AIC = -12533.56

ARMA(2,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	3.418e-01	3.169e-01	1.079	0.2807
ar2	-4.131e-02	2.190e-02	-1.887	0.0592 .
ma1	-3.077e-01	3.167e-01	-0.972	0.3312
intercept	-9.649e-05	2.422e-04	-0.398	0.6903

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:

sigma^2 estimated as 0.0002780, Conditional Sum-of-Squares = 0.65, AIC = -12532.05

ARMA(1,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	3.116e-01	3.387e-01	0.920	0.3575
ma1	-2.779e-01	3.384e-01	-0.821	0.4114
ma2	-3.894e-02	2.156e-02	-1.806	0.0709 .
intercept	-9.551e-05	2.399e-04	-0.398	0.6906

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 0.0002781, Conditional Sum-of-Squares = 0.65, AIC = -12531.84

ARMA(2,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	0.6184105	0.0029575	209.101	<2e-16 ***
ar2	-0.9970848	0.0026396	-377.741	<2e-16 ***
ma1	-0.6285554	0.0056884	-110.498	<2e-16 ***
ma2	0.9835736	0.0061531	159.850	<2e-16 ***
intercept	-0.0002389	0.0004541	-0.526	0.599

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 0.0002684, Conditional Sum-of-Squares = 0.63, AIC = -12613.18

ARMA(3,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	0.6242421	0.0241984	25.797	< 2e-16 ***
ar2	-0.9984609	0.0170624	-58.518	< 2e-16 ***
ar3	0.0142178	0.0218971	0.649	0.51614
ma1	-0.6187572	0.0129956	-47.613	< 2e-16 ***
ma2	0.9577709	0.0214750	44.599	< 2e-16 ***
intercept	-0.0014505	0.0004536	-3.198	0.00139 **

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 0.0002696, Conditional Sum-of-Squares = 0.63, AIC = -12600.41

ARMA(2,3)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	0.6147229	0.0044402	138.446	< 2e-16 ***
ar2	-0.9954027	0.0047228	-210.767	< 2e-16 ***
ma1	-0.6830672	0.0213353	-32.016	< 2e-16 ***
ma2	1.0172130	0.0160185	63.502	< 2e-16 ***
ma3	-0.0715387	0.0202765	-3.528	0.000418 ***
intercept	-0.0020278	0.0004366	-4.645	3.40e-06 ***

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 0.0002718, Conditional Sum-of-Squares = 0.64, AIC = -12581.67

ARMA(3,3)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
ar1	-0.364239	0.023196	-15.70	<2e-16 ***
ar2	-0.364899	0.018813	-19.40	<2e-16 ***
ar3	-1.049142	0.016748	-62.64	<2e-16 ***
ma1	0.431577	0.018495	23.34	<2e-16 ***
ma2	0.471230	0.020837	22.61	<2e-16 ***
ma3	0.881686	0.032134	27.44	<2e-16 ***
intercept	0.057197	0.001991	28.72	<2e-16 ***

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Fit:
sigma^2 estimated as 0.0003545, Conditional Sum-of-Squares = 1.83, AIC = -11956.36

APPENDIX 4

Table 6. Normality tests

H_0: normally distributed
H_1:: non-normally distributed

5% significance level critical W value 0.947

If tested W value > critical value, then H0 is true.

Shapiro-Wilk normality test of Euribor residuals

data: eurresi
W = 0.5003, p-value < 2.2e-16

Shapiro-Wilk normality test of T-Bill residuals

data: tbillresi
W = 0.7845, p-value < 2.2e-16

Shapiro-Wilk normality test of Euro-US dollar exchange rate residuals

data: eurusdresi
W = 0.9929, p-value = 2.805e-09

APPENDIX 5

Table 7. Plausible Euribor GARCH models

GARCH(1,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
a0	1.037e-06	2.347e-08	44.19	<2e-16 ***
a1	1.710e+00	1.636e-02	104.51	<2e-16 ***
b1	5.838e-01	3.541e-03	164.84	<2e-16 ***

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:
Jarque Bera Test

data: Residuals
X-squared = 2913643, df = 2, p-value < 2.2e-16

Box-Ljung test

H_0: no autocorrelation in squared residuals, no ARCH/GARCH effect in residuals
H_1: autocorrelation present in squared residuals, ARCH/GARCH effect present in residuals

critical X-squared value: 3.84 with 95 % confidence level and df = 1

data: Squared.Residuals
X-squared = 0.0398, df = 1, p-value = 0.8418

GARCH(2,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
a0	1.618e-06	3.558e-08	45.48	<2e-16 ***
a1	2.693e+00	2.211e-02	121.80	<2e-16 ***
b1	6.176e-02	3.230e-03	19.12	<2e-16 ***
b2	3.110e-01	5.052e-03	61.56	<2e-16 ***

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:
Jarque Bera Test

data: Residuals
X-squared = 1447109, df = 2, p-value < 2.2e-16

```
        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.0841, df = 1, p-value = 0.7718

GARCH(2,3)

Coefficient(s):
       Estimate     Std. Error    t value      Pr(>|t|)
a0     1.989e-05    5.735e-07     34.674       < 2e-16 ***
a1     1.299e-01    2.055e-02     6.324        2.54e-10 ***
a2     1.237e-09    1.369e-02     9.04e-08     1.000000
a3     1.380e-01    1.226e-02     11.252       < 2e-16 ***
b1     8.077e-02    1.169e-02     6.909        4.88e-12 ***
b2     8.508e-02    2.374e-02     3.584        0.000339 ***
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:
        Jarque Bera Test

data:  Residuals
X-squared = 4859896, df = 2, p-value < 2.2e-16


        Box-Ljung test

data:  Squared.Residuals
X-squared = 0.0079, df = 1, p-value = 0.9294
```

Table 8. Plausible Exchange rate GARCH models

GARCH(2,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
a0	2.590e-05	7.557e-06	3.427	0.000611 ***
a1	1.902e-03	1.780e-02	0.107	0.914917
a2	7.626e-02	2.583e-02	2.952	0.003157 **
b1	1.858e-01	1.677e-01	1.108	0.267970
b2	5.465e-10	1.810e-01	3.02e-09	1.000000

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:
Jarque Bera Test

data: Residuals
X-squared = 66.7842, df = 2, p-value = 3.109e-15

Box-Ljung test

data: Squared.Residuals
X-squared = 1.8601, df = 1, p-value = 0.1726

GARCH(1,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
a0	1.895e-05	4.141e-06	4.576	4.75e-06 ***
a1	8.979e-10	1.688e-02	5.32e-08	1.0000
a2	1.156e-01	2.929e-02	3.946	7.94e-05 ***
b1	3.528e-01	1.271e-01	2.776	0.0055 **

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:
Jarque Bera Test

data: Residuals
X-squared = 66.3654, df = 2, p-value = 3.886e-15

Box-Ljung test

data: Squared.Residuals
X-squared = 1.6913, df = 1, p-value = 0.1934

GARCH(3,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
a0	1.116e-05	2.723e-06	4.101	4.12e-05 ***
a1	7.740e-03	1.688e-02	0.459	0.64657
a2	1.543e-01	3.259e-02	4.735	2.19e-06 ***
b1	3.530e-01	1.239e-01	2.849	0.00439 **
b2	1.131e-13	1.245e-01	9.09e-13	1.00000
b3	1.755e-01	1.098e-01	1.598	0.11001

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:

Jarque Bera Test

data: Residuals
X-squared = 69.9461, df = 2, p-value = 6.661e-16

Box-Ljung test

data: Squared.Residuals
X-squared = 2.919, df = 1, p-value = 0.08754

Table 9. Plausible T-Bill GARCH models

GARCH(1,1)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
a0	9.016e-07	1.486e-07	6.068	1.30e-09 ***
a1	2.115e-01	9.078e-03	23.299	< 2e-16 ***
b1	8.307e-01	5.741e-03	144.688	< 2e-16 ***

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:
Jarque Bera Test

data: Residuals
X-squared = 3217.648, df = 2, p-value < 2.2e-16

Box-Ljung test

data: Squared.Residuals
X-squared = 0.0067, df = 1, p-value = 0.9346

GARCH(1,2)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
a0	9.844e-07	1.676e-07	5.873	4.28e-09 ***
a1	2.342e-01	2.164e-02	10.822	< 2e-16 ***
a2	3.148e-11	2.330e-02	1.35e-09	1
b1	8.168e-01	7.317e-03	111.630	< 2e-16 ***

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:
Jarque Bera Test

data: Residuals
X-squared = 3186.366, df = 2, p-value < 2.2e-16

Box-Ljung test

data: Squared.Residuals

X-squared = 0.0179, df = 1, p-value = 0.8935

GARCH(3,2)

Coefficient(s):

	Estimate	Std. Error	value	Pr(>|t|)
a0	1.841e-06	3.630e-07	5.071	3.95e-07 ***
a1	2.788e-01	2.034e-02	13.703	< 2e-16 ***
a2	1.531e-01	4.630e-02	3.306	0.000945 ***
b1	1.293e-08	1.523e-01	8.49e-08	1.000000
b2	3.220e-01	1.003e-01	3.211	0.001322 **
b3	3.296e-01	3.975e-02	8.293	< 2e-16 ***

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:
Jarque Bera Test

data: Residuals
X-squared = 2636.168, df = 2, p-value < 2.2e-16

Box-Ljung test

data: Squared.Residuals
X-squared = 0.2102, df = 1, p-value = 0.6466

GARCH(2,3)

Coefficient(s):

	Estimate	Std. Error	t value	Pr(>|t|)
a0	1.671e-06	2.644e-07	6.319	2.63e-10 ***
a1	1.173e-01	1.200e-02	9.774	< 2e-16 ***
a2	1.333e-01	1.456e-02	9.156	< 2e-16 ***
a3	9.292e-02	1.844e-02	5.039	4.69e-07 ***
b1	3.367e-13	4.886e-02	6.89e-12	1
b2	7.127e-01	4.284e-02	16.639	< 2e-16 ***

Signif. codes: 0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Diagnostic Tests:
Jarque Bera Test

data: Residuals
X-squared = 4092.811, df = 2, p-value < 2.2e-16

Box-Ljung test

data: Squared.Residuals
X-squared = 4.8296, df = 1, p-value = 0.02798

APPENDIX 6

Table 10. Cointegration test

Phillips-Ouliaris Cointegration Test

data: cbind(Euribor, Tbill)
Phillips-Ouliaris demeaned = -4.5618, Truncation lag parameter = 23, p-value = 0.15